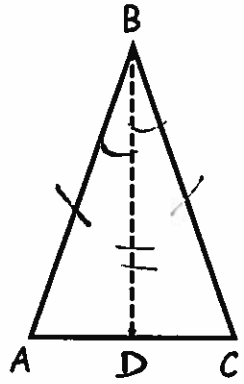
Isosceles Triangle Theorem

Given: $\overline{AB} \cong \overline{BC}$ [Triangle ABC is Isosceles]

Prove: $\angle A \cong \angle C$ [Base Angles are Congruent]

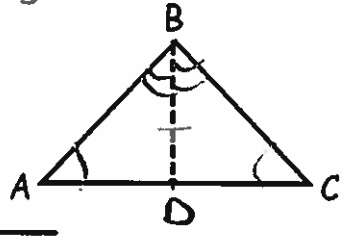


Statements	Reasons
1 $\overline{AB} \cong \overline{BC}$	Given
2 Draw $\overline{BD}$ as the bisector of $\angle B$.	Every $\angle$ has a bisector
3 $\angle ABD \cong \angle CBD$	Def. of $\angle$ bisector
4 $\overline{BD} \cong \overline{BD}$	Refl. Prop of $\cong$
5 $\triangle ABD \cong \triangle CBD$	SAS $\cong$ Post.
6 $\angle A \cong \angle C$	CPCTC

Converse of the Isosceles Triangle Theorem

Given: $\angle A \cong \angle C$ [Base Angles are Congruent]

Prove: $\overline{AB} \cong \overline{CB}$ [Triangle ABC is Isosceles]



Statements	Reasons
1 $\angle A \cong \angle C$	Given
2 Draw $\overline{BD}$ as the $\angle$ bisector of $\angle B$	Every $\angle$ has a bisector.
3 $\angle ABD \cong \angle CBD$	Def. of $\angle$ bisector
4 $\overline{BD} \cong \overline{BD}$	Refl. Prop of $\cong$
5 $\triangle ABD \cong \triangle CBD$	AAS $\cong$ Thm
6 $\overline{AB} \cong \overline{CB}$	CPCTC

Base Angles Theorem

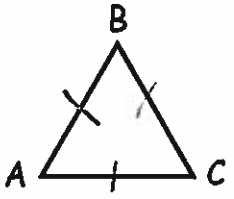
Two sides of a triangle are congruent if and only if the angles opposite the sides are congruent.

Corollary 1:

An equilateral triangle is equiangular.

Given: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$ [The triangle is equilateral.]

Prove: $\angle A \cong \angle B \cong \angle C$ [The triangle is equiangular.]



	Statements	Reasons
1	$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Given
2	$\angle A \cong \angle C$, $\angle A \cong \angle B$	Base $\angle$s Thm
3	$\angle A \cong \angle B \cong \angle C$	Trans. Prop. of $\cong$

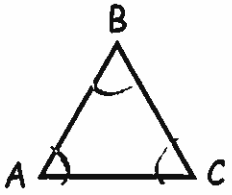
Equilateral $\rightarrow$ Equiangular

Corollary 4:

An equiangular triangle is equilateral.

Given: $\angle A \cong \angle B \cong \angle C$ [The triangle is equiangular.]

Prove: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$ [The triangle is equilateral.]



	Statements	Reasons
1	$\angle A \cong \angle B \cong \angle C$	Given
2	$\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{BC}$	Base $\angle$s Thm
3	$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Trans. Prop. $\cong$

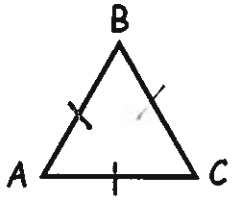
Equiangular → Equilateral

Corollary 2:

An equilateral triangle has three 60° angles.

Given: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$ [The triangle is equilateral.]

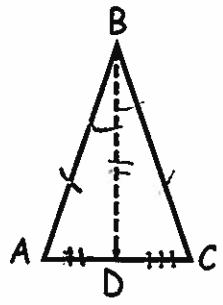
Prove: $m\angle A = m\angle B = m\angle C = 60°$



	Statements	Reasons
1	$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Given
2	$\angle A \cong \angle B \cong \angle C$	Equilateral → Equiangular
3	$m\angle A = m\angle B = m\angle C = 60°$	Equiangular → 3 60° $\angle$s
4		
5		
6		
7		

Corollary 3:

The bisector of the vertex of an isosceles triangle is perpendicular to the base at its midpoint.



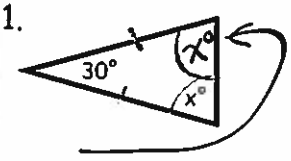
Given: $\overline{AB} \cong \overline{BC}$, $\overline{BD}$ bisects $\angle B$

Prove: $\overline{BD} \perp \overline{AC}$, D is the midpoint of $\overline{AC}$

Statements	Reasons
1 $\overline{AB} \cong \overline{BC}$, $\overline{BD}$ bisects $\angle B$	Given
2 $\angle ABD \cong \angle CBD$	Def of $\angle$ bisector
3 $\overline{BD} \cong \overline{BD}$	Refl. Prop of $\cong$
4 $\triangle ABD \cong \triangle CBD$	SAS $\cong$ Post
5 $\overline{AD} \cong \overline{CD}$, $\angle ADB \cong \angle CDB$	CPCTC
6 D is the midpoint of $\overline{AC}$	Def. of midpt.
7 $\overline{BD} \perp \overline{AC}$	Lines form $\cong$ adj $\angle$s $\rightarrow \perp$ lines

Find the value of x. Justify all answers/equations.

1.



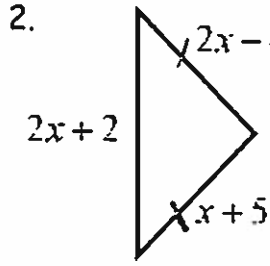
① [Base $\angle$s Thm]

② $2x + 30 = 180$ [$\triangle$ Sum Thm]

$2x = 150$

$x = 75$

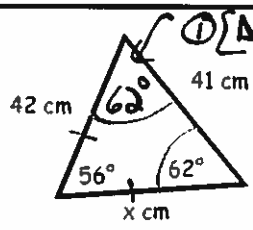
2.



$x + 5 = 2x - 4$ [Def. of $\cong$ segs]

$x = 9$

3.



① [$\triangle$ Sum Thm]

② $x = 42$ [Base $\angle$s Thm]